

Time machines with non-compactly generated Cauchy horizons and “handy singularities”.

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It is 10 years now that time machines (TM's) are intensively studied, but the main question, of *whether or not TM's can be created* remains still unanswered.

Most investigations so far have centered on TM's with compactly generated Cauchy horizons (compact TM's, or CTM's, for brevity) [1]. It has been understood that creation of such TM's is connected with at least two serious problems:

(1) There are some “dangerous” null geodesics in the causal regions of CTM's. A photon propagating along such a geodesic would return infinitely many times (each time blue-shifted) in the vicinity of the Cauchy horizon [2]. This suggests that quantum effects could prevent the creation of the TM. It is still not clear whether they will [3, 4], but they might.

(2) Creation of a CTM inevitably involves violations of the Weak Energy Condition. In this connection it is common to refer to quantum effects, but here again some restrictions exist [5].

There is a little hope that any of these issues will be completely clarified in the foreseen future, since they both involve QFT in curved background which has a lot of its own unsolved problems.

It well may be, however, that actually we need not clarify them. Indeed, *why must we bound ourselves to compact TM's?* The only answer I met in the literature is that noncompactness implies that either infinity or a singularity [6] are involved. So, “extra unpredictable information can enter the spacetime” [1] and we can no more completely control such a TM. This

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is true, indeed, but the point is that this in no way is the distinctive feature of “noncompact” TM’s (NTM’s). In **any** time machine we encounter extra unpredictable information as soon as we intersect the Cauchy horizon (by its very definition) and so for **none** of TM’s the evolution can be completely controlled from the initial surface.

So, we conclude that NTM’s are not a bit worse than CTM’s and preference given to the latter is a sheer matter of tradition [6]. Meanwhile, the example of the Deutsch-Politzer spacetime [7] (DPS) shows that to create an NTM we need neither “dangerous geodesics”, nor “exotic matter”. (Of course the problem remains of how *to cause* a spacetime to evolve in the appropriate manner but as noted above this does not depend on compactness.)

The DPS is obtained as follows. A cut is made along a spacelike segment (i. e. disk D^1) on the Minkowski plane. A copy of the cut is made to the past from the original one. The boundaries of the segments (i. e. 4 points, or two copies of S^0) are removed from the spacetime and the banks of the cuts are glued, the upper bank of each cut is glued to the lower bank of the other cut. The four removed points cannot be returned back and form thus irremovable singularities.

What enables us to render the abovementioned theorems harmless in the case of the DPS is the presence of these quite specific singularities and what makes them so handy is the following

1. Unlike Misner-type singularities, they make the relevant region non-compact,
2. They are absolutely mild (i. e. all curvature scalars are bounded) and so there is no need to invoke quantum gravity to explore these singularities,
3. And they are of laboratory, rather than of cosmological nature. That is they are confined in such a region R of the spacetime M that $M - R =$ (a “good” spacetime) $-$ (a compact set).

All the above suggests that singularities possessing these properties (we shall call them *handy singularities*) are worth studying.

Twenty years ago Ellis and Schmidt [8] constructed several singularities satisfying (1) and (2), but not (3). Besides, those singularities occurring in Minkowski spaces quotiented by discrete isometries were too symmetric, which was interpreted as instability.

Recently an n -dimensional analog of the DPS was obtained [9] by the replacement:

$$D^1 \rightarrow D^{n-1}, \quad S^0 \rightarrow S^{n-2} \quad (1)$$

in the procedure described above. So, we know that n -dimensional handy singularities exist, but that is all we know.

And now I would like to propose a simple trick (just generalizing that from [9]) for constructing quasiregular singularities (including the previously known), which yields at the same time a variety of handy singularities.

1. Take a region R in an n -dimensional spacetime M and an $(n-1)$ -dimensional submanifold $S \subset R$ bounded by a closed $(n-2)$ -dimensional submanifold C . Denote by \widetilde{M} the universal covering of $M - C$ and by π the natural projection $\widetilde{M} \mapsto M - C$. Let p be the “natural embedding” $p : M - C \mapsto \widetilde{M}$, $p = \pi^{-1}$.

2. Now make a cut along S in M , that is consider $M_S \equiv \overline{p(\widetilde{M})}$. Note that $S_0 \equiv M_S - p(M)$ is a double covering of S : $\pi(S_0) = S$. If S is orientable, S_0 is just a disjoint union of two copies of S (the two “banks” of the cut): S_+ and S_- . The projection π induces a nontrivial isometry $\sigma : S_0 \mapsto S_0$ enabling one to return to M from M_S by “gluing the banks”. Namely, $M_S/\sigma = M - C$

3. As the third step take an isometry $\eta : R \mapsto R'$, and repeat the above procedure with M replaced by M_S and S replaced by $S' \equiv \eta(S)$. The resulting space $M_{SS'}$ is M with two cuts made (along S and along S'), each taken with its “banks”. The desired spacetime N can be obtained now by the appropriate identification:

$$N \equiv M_{SS'}/\xi \quad (2)$$

where $\xi \equiv \sigma \circ \eta$ (rigorously speaking instead of η we should have written some η' in (2), where η' is the continuous extension of $p \circ \eta$ on $M_{SS'}$, we shall neglect such subtleties for simplicity of notation). In the orientable case (2) simply means that we must glue S_{\pm} to S_{\mp} .

When η is nontrivial, we cannot return C back and N contains thus a handy singularity “in the form of” C . We can also use in (2) any other isometry $\xi \neq \sigma$ (see example (d) below).

If S was chosen to be a disc D^{n-1} in the Minkowski space and η to be a translation, we obtain the DPS (cf. (1)), but being applied to different M , S , and η the same procedure will give us a lot of quite different “handy singularities”.

3-dimensional examples. In what follows M for simplicity is taken to be a flat space \mathbb{R}^3 and z, ρ, ϕ are the cylinder coordinates in it.

(a) Let C be an arbitrary knot (with S being its Seifert surface) and η be a translation. Then N is what is called a “loop-based wormhole” in [10].

(b) Let S be a disk $z = 0$, $\rho < \rho_0$ and η be a rotation $\phi \rightarrow \phi + \phi_0$ (note that $S = S'$ thus). In this case N has quite a curious structure. It is diffeomorphic to $M - C$, any simply connected region of N is isometric to some region of $M - C$ and vice versa, and still globally they are not isometric. Similarly to the conical case one can think of N as a space \mathbb{R}^3 having delta-like Riemannian tensor with support in C (in contrast to M , where the Riemannian tensor is zero even in the distributional sense).

(c) Take a rectangle strip. Turn one of its ends through $n\pi$ and then glue it to the other (so that a cylinder is obtained if $n = 0$ and the Möbius band, if $n = 1$). Take the resulting surface for S . S can be specified by condition

$$0 < r < 1/2, \quad \theta = \pm n\phi$$

Here r and θ are new coordinates. For any point A , $r(A)$ is defined to be the distance from A to a and $\theta(A)$ to be the angle between the z -axis and the direction (Aa) , where a is the point $z(a) = 0$, $\rho(a) = 1$, $\phi(a) = \phi(A)$. For a (local) isometry one can take $\eta: \phi \rightarrow \phi + \phi_0$, $\theta \rightarrow \theta + n\phi_0$. When $n = 3$, C is a knot (trefoil).

(d) Now change \mathbb{R}^3 to $\mathbb{R}^3 \setminus \{0\}$ in example (b) and take ξ in (2) to be the reflection $\mathbf{r} \rightarrow -\mathbf{r}$. Thus obtained N has a singularity generated by the circle C and two more corresponding to the removed $\{0\}$ (it also cannot be returned back). The latter though being handy singularities have locally the same structure as a singularity considered in [8].

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